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ASCENDING DETOUR RADIUS DECOMPOSITION OF A GRAPH

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ABSTRACT

The detour distance D(u, v) between two vertices u and v in a connected graph G from u to v is defined as the length of a longest u – v path in G. The detour eccentricity $e_D(v)$ of v is the maximum detour distance from v to a vertex of G. The minimum detour eccentricity among the vertices of G is the detour radius, rad_D G or $R(G)$ofG .A decomposition of a graph G is a collection π of edge-disjoint sub graphs$G_1, G_2, \dots G_n$of G such that every edge of G belongs to exactly one $G_i(1 \le i \le n)$. The decomposition $\pi = \{G_1, G_2, \cdots, G_n\}$ be a decomposition of G is called an ascending detour radius decomposition of G if $R(G_i) < R(G_{i+1})$; $i = 1$ to n. In this paper we studied the connected graphs which admits ascending detour radius decomposition and which does not admit ascending detour radius decomposition

KEYWORDS: Detour, Detour Radius, Decomposition, Ascending Detour Radius Decomposition

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1. INTRODUCTION

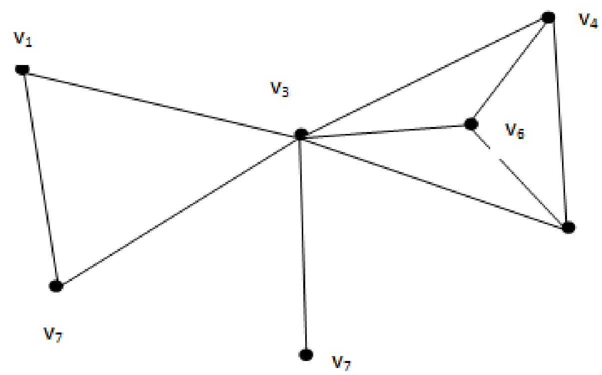
By a graph $G = (V, E)$, we mean a finite undirected connected graph without loops or multiple edges. The order and size of *G* are denoted by *n* and *m* respectively. For basic graph theoretic terminology, we refer to Harary [2]. The *distance d* (*u, v*) between two vertices *u* and *v* in a connected graph *G* is the length of a shortest *u*–*v* path in *G*. An *u*–*v* path of length *d* (*u*, *v*) is called an *u*–*v* geodesic. A vertex *x* is said to lie on a *u*-*v* geodesic *P* if *x* is a vertex of *P* including the vertices *u* and *v*.The *detour distance D* (*u*, *v*) between two vertices *u* and *v* in a connected graph *G* from *u* to *v* is defined as the length of a longest *u* – *v* path in *G*. A *u* – *v* path of length *D* (*u*, *v*) is called a *u* – *v detour*. A vertex *x* is said to lie on a *u*-*v* detour *P* if *x* is a vertex of *P* including the vertices *u* and *v*. The *detour eccentricity* $e_D(v)$ of *v* is the maximum detour distance from *v* to a vertex of *G*. The minimum detour eccentricity among the vertices of *G* is the *detour radius*, $rad_D(G)$or $R(G)$of *G* and the maximum detour eccentricity is its *detour diameter*, $diam_D(G)$or $D(G)$ of *G*. Each vertex in *V* at which the detour eccentricity function is minimized is called a *detour central vertex* of *G* and the set of all central vertices of *G* is called the detour *center* of *G* and is denoted by $Z_D(G)$. Two vertices *u* and *v* are *detour antipodal* vertices if $D(u, v) = diam_D(G)$.This concept were studied in [3,4] .A decomposition of a graph G is a collection π of edge-disjoint sub graphs$G_1, G_2, \dots G_n$of G such that every edge of G belongs to exactly one $G_i(1 \le i \le n)$. This concept were studied in [1,5].For any set *M* of vertices of *G*, the *induced subgraph*<*M*> is the maximal subgraph of *G* with vertex set *M*. The bi-star graph $B_{m,n}$is a tree with radius3.A fan graph $f_{1,n-1}$ is $K_1 + P_n (n \ge 2)$. The friendship graph F_n can be constructed by joining n copies of the cycle C_3 with a common vertex. i.e., every

two vertices have exactly one neighbour in common are exactly the friendship graphs.

2. ASCENDING DETOUR RADIUS DECOMPOSITION OF A GRAPH

Definition2.1 The decomposition$\pi = \{G_1, G_2, \cdots, G_n\}$ is called an *ascending detourradius decomposition* of G if $R(G_i) < R(G_{i+1}); i = 1\ to\ n.$

Example2.2 For the graph Ggiven in Figure 2.1, a decomposition of G is given in Figure 2.1(a). Since$R(G_1) = 1$, $R(G_2) = 2$ and $R(G_3) = 3.\pi = \{G_1, G_2, G_3\}$ is anascending detour radius decomposition of G.



G

Figure 2.1

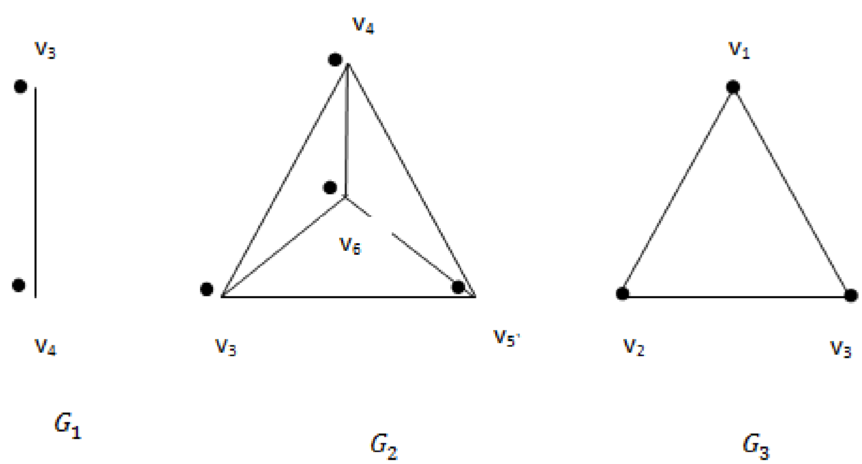


Figure 2.1(a)

Remark 2.3 There can be more than one ascending detour radius decomposition of G.For the graph G given in Figure 2.2,$\pi_1 = \{G_1, G_2\}$ and $\pi_2 = \{G_3, G_4\}$ are two ascending detour radius decompositions of G.

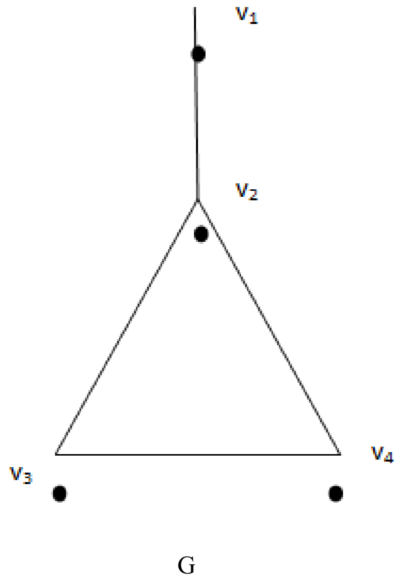


Figure 2.2

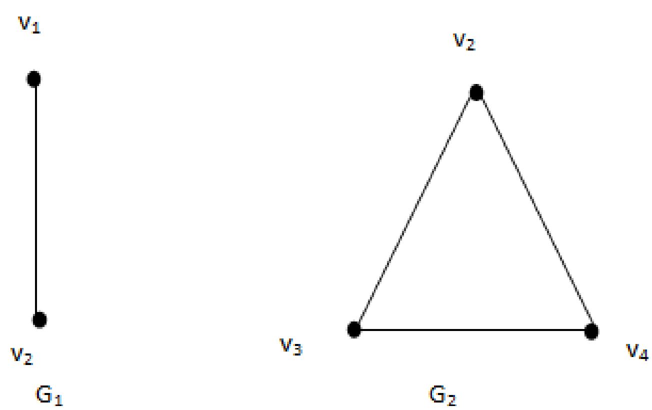


Figure 2.2(a)

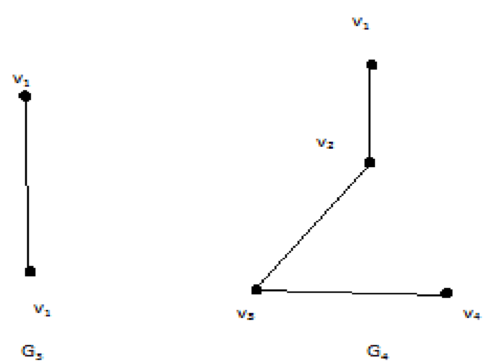


Figure 2.2 (b)

In the following, wedetermine the graph which admitsascending detour radius decomposition and which does not admits ascending detour radius decomposition.

Theorem2.4 The star graph doesn't admitascending detour radius decomposition.

Proof: The star $G = K_{1,n-1}$ has detour radius $R = 1$.since every sub graph of G has detour radius$R = 1$, the star graph doesn't admits an ascending detour radius decomposition of$G$.

Theorem2.5 The cycle C_n admits ascending detour radius decomposition if and only if $n \geq 4$.

Proof: Let $n \geq 4$ and let $C_n: v_1, v_2, v_3, \cdots, v_n, v_1$be the cycle.Let$S_1 = \{v_1, v_2\}$, $S_2 = \{v_2, v_2, \cdots, v_n\}andG_1 =< S_1 >andG_2 =< S_2 >$.Then $\pi = \{G_1, G_2\}$ is anascending detour radius decomposition of G.Hence G admits an ascending detour radius decomposition.

Conversely let C_n admits ascending detour radius decomposition. We prove that $n \geq 4$.Suppose $n = 3$. Then any subgraph of G has detour radius$R = 1$.Hence C_3 doesn't admitascending detour radius decomposition. Therefore$n \geq 4$.

Theorem 2.6 The path P_n admits ascending detour radius decomposition if and only if $n \geq 5$.

Proof: Let$n \geq 5$. Let $S_1 = \{v_1, v_2\}$, $S_2 = \{v_2, v_2, \cdots, v_n\}$ and $G_1 =< S_1 >$,$G_2 =< S_2 >$.Then $\pi = \{G_1, G_2\}$ is anascending detour radius decomposition of G.Hence G admits an ascending detour radius decomposition.

Conversely let P_n admits ascending detour radius decomposition. We prove that $n \geq 5$.Suppose $n \leq 4$. Then any subgraph of G has detour radius $R = 1$.Hence P_n doesn't admits an ascending detour radius decomposition for $n \leq 4$.Therefore $n \geq 5$.

Theorem 2.7 The Bi-star graph $B_{m,n}$ admits an ascending detour radius decomposition if and only if $m \geq 2$ and $n \geq 1$ or $m \geq 1$and $n \geq 2$.

Proof: Let u, v be the cut vertices of G. Let $uu_1, uu_2, \cdots, uu_n, vv_1, vv_2, \cdots, vv_m$ be the end edges of G.Let us assume $m \geq 2$ and $n \geq 1$. Let $S_1 = \{v_1, u, v, u_1, u_2 \cdots, u_n\}$and $S_2 = \{v_2, v_3, \cdots, v_m, v\}$.*Let* $G_1 =< S_1 >$and$G_2 =< S_2 >$.Then $R(G_1) = 2$ and$R(G_2) = 1$.Therefore$\pi = \{G_1, G_2\}$ is anascending detour radius decomposition of G so that$B_{m,n}$ admits an ascending detour radius decomposition for $m \geq 2$ and $n \geq 1$.Similarly for $m \geq 1$ and $n \geq 2$, we can prove $B_{m,n}$ admits an ascending detour radius decomposition.

Conversely, let $B_{m,n}$ admits an ascending detour radius decomposition .We prove that $m \geq 2$ and $n \geq 1$ or $m \geq 1$ and $n \geq 2$.Suppose that $m = 1$and $n = 1$.Since$R(B_{m,n}) = 1$, anysub graph of $B_{m,n}$ has detour radius$R = 1$. Therefore $B_{m,n}$ doesn't admitascending detour radius decomposition. Which is a contradiction, Therefore$m \geq 2$ and $n \geq 1$ or $m \geq 1$and$n \geq 2$.

Theorem 2.8 The complete graph K_n admits ascending detour radius decomposition if and only if $n \geq 4$.

Proof: Let $n \geq 4$. Let e be an edge of K_n such that $G_1 = e$ and $G_2 = K_n - e$.Then $R(G_1) = 1$ and $R(G_2) \geq 3$.Hence $\pi = \{G_1, G_2\}$ is anascending detour radius decomposition of G. Therefore K_n admits an ascending detour radius decomposition.

Conversely, let K_n admits an ascending detour radius decomposition .We have to prove $n \geq 4$.Suppose that $n = 3$. Then asub graph of K_n has detour radius $R = 1$.Therefore $K_n$doesn't admitsan ascending detour radius decomposition. Which is a contradiction, Therefore $n \geq 4$.

Theorem 2.9 The Bipartitegraph $K_{m,n}$ admits an ascending detour radius decomposition if and only if $m \geq 2$and$n \geq 2$.

Proof: Let us assume $m \geq 2$ and $n \geq 2$.Let $U = \{u_1, u_2, \cdots, u_m\}$ and $V = \{v_1, v_2, \cdots, v_n\}$ be the partitions of G.Let $G_1 = u_1v_1$ and $G_2 = G - u_1v_1$.Then $R(G_1) = 1$ and$R(G_2) \geq 3$Therefore $\pi = \{G_1, G_2\}$ is anascending detour radius decomposition of G and so $K_{m,n}$ admits an ascending detour radius decomposition.

Conversely, let $K_{m,n}$ admits an ascending detour radius decomposition .We prove that $m \geq 2$ and $n \geq 2$.Suppose that $m = 1$ and $n \geq 1$ or $m \geq 1$ and $n = 1$.Then G is a star. Then by Theorem 2.4,Gdoesn't admitascending detour radius decomposition. Which is a contradiction, Therefore $m \geq 2$and $n \geq 2$.

Theorem 2. 10 The fan graph $G = f_{1,n-1}$admitsascending detour radius decomposition if and only if$n \geq 3$.

Proof: Let us assume $n \geq 3$.Let x be the centre vertex and $v_1, v_2, \cdots, v_{n-1}$ be the remaining vertices of the fan graph $f_{1,n-1}$. Let $S_1 = xv_1$and $S_2 = \{xv_2, xv_3, \cdots, xv_{n-1}\}$ such that $G_1 =< S_1 >$and$G_2 =< S_2 >$Then $R(G_1) = 1$ and $R(G_2) \geq 3$.Therefore $\pi = \{G_1, G_2\}$ is anascending detour radius decomposition of$G$ and so $f_{1,n-1}$ admits an ascending detour radius decomposition.

Conversely, let $f_{1,n-1}$ admits ascending detour radius decomposition. We have to prove $n \geq 3$.Suppose that $n = 2$.Then asub graph of $f_{1,n-1}$has detour radius $R = 1$.Therefore $f_{1,n-1}$doesn't admitsan ascending detour radius decomposition. Which is a contradiction, Therefore$n \geq 3$.

Theorem 2. 11 The Friendship graph$G = K_1 + \cup\, mK_2$,$m \geq 2$ admits ascending detour radius decomposition if and only if$m \geq 2$.

Proof: Let$m \geq 2$. Let e be an edge of $K_1 + \cup\, mK_2$ such that $G_1 = e$and $G_2 = G - e$.Then $R(G_1) = 1$ and $R(G_2) \geq 2$.Hence $\pi = \{G_1, G_2\}$ is anascending detour radius decomposition of$G$. Therefore $K_1 + \cup\, mK_2$ admits an ascending detour radius decomposition.

Conversely, let $K_1 + \cup\, mK_2$admits ascending detour radius decomposition. We have to prove $m \geq 2$.Suppose that $m = 1$.Then G is K_3.Then any sub graph of K_3 has detour radius $R = 1$

.Therefore $K_1 + \cup\, mK_2$doesn't admitan ascending detour radius decomposition. Which is a contradiction, Therefore$m \geq 2$.

Theorem 2. 12 The wheel graph $G = W_{1,n-1}$admits an ascending detour radius decomposition if and only if $n \geq 4$.

Proof: Let$n \geq 4$. Let e be an edge of G such that $G_1 = e$ and $G_2 = G - e$.Then $R(G_1) = 1$and $R(G_2) \geq 2$.Hence $\pi = \{G_1, G_2\}$ is anascending detour radius decomposition of$G$. Therefore $W_{1,n-1}$admits an ascending detour radius decomposition.

Conversely, let $W_{1,n-1}$admits an ascending detour radius decomposition .We have to prove $n \geq 4$.Suppose that $n = 3$.Then a sub graph of $W_{1,n-1}$has detour radius$R = 1$.Therefore $W_{1,n-1}$doesn't admit an ascending detour radius decomposition, which is a contradiction, Therefore$n \geq 4$.

CONCLUSIONS

In this paper we studied the connected graph which admitsascending detour radius decomposition and which doesn't admits ascending detour radius decomposition. In future work we will study the ascending detour radius decomposition number of a connected graph.

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